Taurus: Towards a Unified Force Representation and Universal Solver for Graph Layout

(Supplemental Material)

Mingliang Xue, Zhi Wang, Fahai Zhong, Yong Wang
Oliver Deussen, and Yunhai Wang

Abstract—We provide more details of our evaluation results in this supplemental material. Section 1 shows the quantitative results of our experiments, Section 2 presents the visualisation results from the experiments, and Section 3 further clarifies the results and running times for the 10 large graphs.

1 Quantitative Results

We have extensively evaluated our approach through different quantitative metrics, including stress error (SE) [3], neighborhood preservation (NP) [10], Crosslessness (CL) [12], Minimum Angle (MA) [8] on all graph layouts and cluster extraction (CE), and cluster distance (CD) [11] on clustered graphs. We investigated six graph drawing techniques, including our balanced stress model, stress model [4], Maxent [3], force-directed placement [2], LinLog [7] and FM^3 [5]. For all the six graph drawing techniques, We compared the implementations using our framework (with an F suffix after the technique name) and the original implementation (except our BSM). Tables 1, 2, 3, and 4 show the detailed statistics of different datasets. Tables 5, 6, 7, 8, 9, and 10 present the detailed results of the metrics SE, NP, CL, MA, CE and CD, respectively. As mentioned in Section 5.1 of the main paper, we have adjusted the normalized metric values to guarantee that a larger value indicates a better performance.

Besides, Table 11 shows the detailed runtime (seconds) of different methods on each graph. We also validate the effectiveness of our augmented SGD solver by comparing it with SGD and BH on these datasets for FDP method. The results of SGD and BH are shown in Columns FDP-SGD and FDP-BH, while the results of our augmented SGD solver are shown in Column FDP-F.

Table 1. The statistics of grid graphs.

	Nodes	Edges	Description and References
grid_100	100	180	2D grid
grid_289	289	544	2D grid
grid_500	500	955	2D grid
grid_800	800	1600	ring grid
grid_1000	1000	2000	ring grid
grid_1200	1200	2400	ring grid
grid_1500	1500	4100	3D grid
grid_1600	1600	3120	2D grid
grid_2000	2000	5500	3D grid
grid_2250	2250	6225	3D grid
grid_2500	2500	4900	2D grid
grid_3000	3000	8350	3D grid
grid_3288	3288	8791	3D grid
grid_3600	3600	7200	ring grid
grid_4800	4800	9452	ring grid

- *Mingliang Xue, Zhi Wang, Fahai Zhong, and Yunhai Wang are with the Department of Computer Science, Shandong University, China. E-mail: {xml95007, wangzizi2020, zhongfahai, cloudseawang}@gmail.com*
- *Yong Wang is with School of Computing and Information Systems, Singapore Management University, Singapore. E-mail: yongwang@smu.edu.sg*
- *Oliver Deussen is with Computer and Information Science, University of Konstanz, Konstanz, Germany. E-mail: oliver.deussen@uni-konstanz.de*
- *Yunhai Wang is the corresponding author*

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Table 2. The statistics of Binary Trees.

	Nodes	Edges	Description and References
btree_129	129	128	binary tree
btree_193	193	192	binary tree
btree_257	257	256	binary tree
btree_321	321	320	binary tree
btree_385	385	384	binary tree
btree_449	449	448	binary tree
btree_513	513	512	binary tree
btree_577	577	576	binary tree
btree_705	705	704	binary tree
btree_769	769	768	binary tree
btree_1025	1025	1024	binary tree
btree_1537	1537	1536	binary tree
btree_2049	2049	2048	binary tree
btree_3073	3073	3072	binary tree
btree_4097	4097	4096	binary tree

Table 3. The statistics of Clustered Graphs.

	Nodes	Edges	Description and References
cluster_198	198	431	cluster graphs with 5 clusters
cluster_378	378	571	cluster graphs with 6 clusters
cluster_596	596	1501	cluster graphs with 10 clusters
cluster_800	800	3065	cluster graphs with 15 clusters
cluster_991	991	2445	cluster graphs with 10 clusters
cluster_1197	1197	3581	cluster graphs with 12 clusters
cluster_1485	1485	3369	cluster graphs with 9 clusters
cluster_1983	1983	4974	cluster graphs with 10 clusters
cluster_1998	1998	6485	cluster graphs with 13 clusters
cluster_2998	2998	11117	cluster graphs with 15 clusters
cluster_3486	3486	9683	cluster graphs with 11 clusters
cluster_3995	3995	14002	cluster graphs with 14 clusters
cluster_4463	4463	11206	cluster graphs with 10 clusters
cluster_4962	4962	12486	cluster graphs with 15 clusters
cluster_4999	4999	19016	cluster graphs with 10 clusters

Table 4. The statistics of Real Graphs.

	Nodes	Edges	Description and References
bio-CE-GT	878	3181	biological data [9]
bio-CE-HT	2194	2688	biological data [9]
bio-celegans	453	2025	biological data [9]
bio-yeast	1458	1948	biological data [9]
bus494	494	586	power network problem [9]
bus662	662	906	power network problem [9]
bus685	685	1282	power network problem [9]
bus1138	1138	1458	power network problem [3, 4]
ca-CSphd	1025	1043	collaboration network [9]
ca-Erdos992	4991	7428	collaboration network [9]
CA-GrQc	4158	13422	collaboration network [6, 12]
fb-pages-food	620	2102	social network [9]
soc-wiki-Vote	889	2914	social network [9]
US_powergrid	4941	6594	US power grid [6, 12]
visbrazil	222	336	collaboration network [6, 12]

Table 5. Normalized stress errors of 13 layout methods on different graphs.

	BSM	SM	SM-F	Maxent	Maxent-F	FDP	FDP-F	LinLog	LinLog-F	FM^3	FM^3-F	FDP-SGD	FDP-BH
cluster_198	0.873	0.89	0.89	0.842	0.845	0.85	0.873	0.785	0.78	0.863	0.867	0.858	0.879
cluster_378	0.934	0.938	0.938	0.921	0.882	0.8	0.865	0.804	0.787	0.899	0.889	0.811	0.802
cluster_596	0.86	0.872	0.872	0.84	0.843	0.847	0.857	0.769	0.766	0.851	0.853	0.844	0.853
cluster_800	0.837	0.849	0.849	0.804	0.804	0.828	0.838	0.775	0.782	0.838	0.833	0.831	0.838
cluster_991	0.858	0.869	0.871	0.842	0.849	0.834	0.838	0.764	0.767	0.854	0.855	0.832	0.849
cluster_1197	0.85	0.861	0.862	0.827	0.836	0.832	0.836	0.767	0.769	0.845	0.847	0.824	0.841
cluster_1485	0.866	0.877	0.877	0.858	0.853	0.823	0.847	0.774	0.745	0.854	0.849	0.803	0.846
cluster_1983	0.86	0.87	0.871	0.846	0.852	0.828	0.837	0.778	0.772	0.839	0.836	0.818	0.831
cluster_1998	0.844	0.857	0.857	0.829	0.835	0.827	0.834	0.779	0.774	0.844	0.846	0.817	0.836
cluster_2998	0.842	0.853	0.854	0.829	0.833	0.827	0.826	0.783	0.784	0.836	0.837	0.818	0.828
cluster_3486	0.864	0.872	0.872	0.855	0.83	0.814	0.825	0.783	0.776	0.844	0.842	0.81	0.806
cluster_3995	0.847	0.858	0.858	0.837	0.836	0.823	0.822	0.784	0.781	0.843	0.839	0.81	0.824
cluster_4463	0.865	0.873	0.873	0.853	0.809	0.823	0.824	0.782	0.784	0.851	0.848	0.817	0.83
cluster_4962	0.861	0.87	0.871	0.853	0.807	0.827	0.824	0.786	0.772	0.832	0.834	0.812	0.824
cluster_4999	0.846	0.855	0.856	0.836	0.837	0.824	0.826	0.778	0.784	0.853	0.847	0.802	0.825
grid_100	0.983	0.984	0.984	0.984	0.984	0.977	0.98	0.924	0.923	0.977	0.973	0.876	0.979
grid_289	0.986	0.986	0.986	0.986	0.986	0.931	0.983	0.781	0.851	0.979	0.977	0.815	0.84
grid_500	0.987	0.987	0.987	0.987	0.986	0.822	0.983	0.801	0.873	0.983	0.982	0.749	0.823
grid_800	0.908	0.918	0.918	0.9	0.911	0.816	0.913	0.78	0.841	0.914	0.916	0.773	0.86
grid_1000	0.889	0.904	0.904	0.87	0.893	0.845	0.899	0.784	0.854	0.901	0.901	0.72	0.782
grid_1200	0.871	0.892	0.892	0.879	0.883	0.792	0.889	0.752	0.785	0.891	0.89	0.76	0.776
grid_1500	0.921	0.928	0.928	0.919	0.916	0.913	0.922	0.778	0.864	0.924	0.923	0.837	0.917
grid_1600	0.988	0.988	0.988	0.988	0.985	0.784	0.958	0.723	0.741	0.986	0.984	0.627	0.807
grid_2000	0.932	0.937	0.937	0.933	0.925	0.872	0.923	0.793	0.843	0.934	0.934	0.827	0.89
grid_2250	0.935	0.936	0.936	0.915	0.928	0.922	0.93	0.839	0.808	0.931	0.93	0.808	0.924
grid_2500	0.988	0.988	0.988	0.988	0.984	0.666	0.937	0.643	0.654	0.986	0.984	0.609	0.729
grid_3000	0.944	0.944	0.944	0.927	0.92	0.839	0.934	0.777	0.817	0.94	0.938	0.776	0.839
grid_3288	0.907	0.921	0.921	0.916	0.915	0.846	0.914	0.772	0.818	0.908	0.903	0.792	0.863
grid_3600	0.834	0.88	0.88	0.873	0.877	0.804	0.836	0.671	0.695	0.871	0.874	0.644	0.754
grid_4800	0.955	0.96	0.96	0.955	0.958	0.697	0.77	0.633	0.581	0.946	0.944	0.543	0.73
btree_129	0.908	0.922	0.921	0.917	0.909	0.808	0.885	0.792	0.793	0.903	0.899	0.798	0.848
btree_193	0.894	0.91	0.91	0.883	0.893	0.798	0.868	0.777	0.804	0.886	0.866	0.763	0.824
btree_257	0.889	0.905	0.906	0.896	0.888	0.806	0.881	0.789	0.808	0.853	0.85	0.758	0.834
btree_321	0.887	0.902	0.901	0.894	0.885	0.796	0.857	0.795	0.784	0.845	0.844	0.748	0.826
btree_385	0.883	0.896	0.897	0.873	0.878	0.782	0.854	0.795	0.757	0.87	0.783	0.751	0.818
btree_449	0.882	0.897	0.896	0.88	0.88	0.776	0.852	0.797	0.768	0.847	0.806	0.747	0.815
btree_513	0.88	0.892	0.893	0.888	0.874	0.775	0.843	0.802	0.768	0.848	0.798	0.746	0.811
btree_577	0.879	0.894	0.893	0.886	0.878	0.778	0.829	0.785	0.758	0.861	0.772	0.744	0.803
btree_705	0.879	0.891	0.891	0.881	0.87	0.777	0.822	0.784	0.763	0.836	0.787	0.745	0.804
btree_769	0.873	0.887	0.887	0.873	0.869	0.779	0.823	0.782	0.759	0.832	0.838	0.749	0.807
btree_1025	0.873	0.884	0.885	0.876	0.863	0.773	0.817	0.778	0.755	0.836	0.796	0.748	0.796
btree_1537	0.869	0.879	0.88	0.869	0.855	0.778	0.814	0.782	0.748	0.838	0.743	0.754	0.798
btree_2049	0.869	0.878	0.878	0.87	0.853	0.776	0.809	0.781	0.745	0.823	0.778	0.757	0.793
btree_3073	0.867	0.873	0.874	0.862	0.849	0.784	0.804	0.779	0.747	0.811	0.741	0.76	0.792
btree_4097	0.866	0.873	0.873	0.86	0.844	0.784	0.8	0.784	0.751	0.805	0.8	0.763	0.79
bio-CE-GT	0.884	0.889	0.894	0.844	0.837	0.863	0.866	0.817	0.775	0.878	0.88	0.839	0.866
bio-CE-HT	0.859	0.869	0.869	0.846	0.854	0.823	0.806	0.804	0.793	0.849	0.849	0.771	0.828
bio-celegans	0.874	0.88	0.88	0.834	0.792	0.844	0.839	0.795	0.725	0.852	0.859	0.829	0.846
bio-yeast	0.876	0.886	0.886	0.862	0.862	0.843	0.841	0.811	0.778	0.868	0.863	0.798	0.855
bus494	0.924	0.933	0.933	0.921	0.898	0.837	0.883	0.798	0.782	0.906	0.895	0.783	0.857
bus662	0.927	0.932	0.932	0.92	0.908	0.838	0.869	0.786	0.801	0.918	0.913	0.784	0.85
bus685	0.963	0.965	0.965	0.955	0.937	0.88	0.902	0.805	0.8	0.928	0.941	0.776	0.867
bus1138	0.934	0.938	0.938	0.931	0.92	0.817	0.858	0.791	0.726	0.902	0.9	0.72	0.839
ca-CSphd	0.923	0.926	0.927	0.913	0.896	0.778	0.741	0.772	0.716	0.896	0.891	0.728	0.814
ca-Erdos992	0.869	0.874	0.874	0.845	0.858	0.845	0.811	0.817	0.805	0.863	0.86	0.785	0.851
CA-GrQc	0.871	0.877	0.878	0.845	0.807	0.844	0.827	0.774	0.788	0.861	0.858	0.764	0.751
fb-pages-food	0.913	0.92	0.92	0.874	0.83	0.847	0.867	0.804	0.703	0.888	0.892	0.8	0.776
soc-wiki-Vote	0.899	0.904	0.904	0.867	0.845	0.855	0.847	0.784	0.768	0.873	0.878	0.836	0.854
us_powergrid	0.939	0.942	0.942	0.916	0.906	0.755	0.774	0.735	0.691	0.89	0.868	0.697	0.789
visbrazil	0.936	0.94	0.94	0.904	0.917	0.913	0.887	0.834	0.79	0.913	0.911	0.887	0.864

Table 6. Normalized neighborhood preservation degrees of 13 layout methods on different graphs.

	BSM	SM	SM-F	Maxent	Maxent-F	FDP	FDP-F	LinLog	LinLog-F	FM^3	FM^3-F	FDP-SGD	FDP-BH
cluster_198	0.368	0.27	0.267	0.176	0.181	0.285	0.311	0.443	0.444	0.358	0.387	0.304	0.382
cluster_378	0.282	0.257	0.253	0.209	0.221	0.205	0.311	0.403	0.428	0.25	0.254	0.253	0.252
cluster_596	0.265	0.156	0.157	0.109	0.115	0.221	0.243	0.382	0.406	0.194	0.205	0.229	0.238
cluster_800	0.297	0.145	0.144	0.095	0.095	0.214	0.216	0.455	0.461	0.172	0.191	0.219	0.227
cluster_991	0.192	0.115	0.104	0.088	0.097	0.179	0.164	0.298	0.31	0.16	0.163	0.161	0.188
cluster_1197	0.223	0.114	0.117	0.088	0.094	0.199	0.183	0.319	0.337	0.164	0.168	0.179	0.204
cluster_1485	0.139	0.081	0.086	0.08	0.084	0.121	0.126	0.245	0.256	0.116	0.117	0.104	0.129
cluster_1983	0.121	0.065	0.067	0.068	0.078	0.11	0.103	0.218	0.23	0.148	0.141	0.157	0.165
cluster_1998	0.183	0.086	0.084	0.072	0.083	0.16	0.155	0.267	0.279	0.1	0.103	0.097	0.112
cluster_2998	0.172	0.071	0.07	0.06	0.077	0.158	0.153	0.252	0.263	0.142	0.138	0.153	0.159
cluster_3486	0.095	0.053	0.054	0.061	0.081	0.091	0.092	0.182	0.186	0.089	0.087	0.085	0.091
cluster_3995	0.127	0.058	0.059	0.057	0.073	0.119	0.113	0.196	0.204	0.112	0.109	0.111	0.118
cluster_4463	0.064	0.037	0.037	0.047	0.056	0.057	0.057	0.153	0.157	0.056	0.056	0.054	0.06
cluster_4962	0.059	0.033	0.034	0.048	0.055	0.053	0.053	0.148	0.151	0.11	0.109	0.11	0.118
cluster_4999	0.12	0.051	0.053	0.055	0.074	0.117	0.115	0.187	0.194	0.055	0.052	0.049	0.056
grid_100	0.991	1	1	1	1	0.763	0.819	0.653	0.648	0.762	0.756	0.537	0.793
grid_289	0.973	1	1	1	1	0.553	0.803	0.406	0.502	0.708	0.783	0.306	0.392
grid_500	0.956	1	1	0.997	0.97	0.325	0.779	0.34	0.416	0.736	0.779	0.228	0.395
grid_800	0.317	0.338	0.336	0.287	0.271	0.171	0.314	0.247	0.232	0.31	0.316	0.156	0.228
grid_1000	0.297	0.291	0.288	0.239	0.241	0.201	0.277	0.225	0.208	0.289	0.293	0.116	0.168
grid_1200	0.285	0.268	0.266	0.249	0.249	0.152	0.27	0.201	0.245	0.261	0.268	0.136	0.163
grid_1500	0.199	0.193	0.194	0.214	0.151	0.218	0.202	0.218	0.277	0.184	0.192	0.155	0.235
grid_1600	0.948	0.999	0.999	0.994	0.826	0.222	0.649	0.239	0.276	0.752	0.748	0.105	0.364
grid_2000	0.2	0.187	0.19	0.21	0.192	0.149	0.206	0.192	0.261	0.183	0.184	0.139	0.198
grid_2250	0.189	0.167	0.173	0.189	0.138	0.189	0.182	0.211	0.255	0.196	0.199	0.128	0.207
grid_2500	0.944	0.998	0.998	0.994	0.809	0.112	0.553	0.224	0.267	0.782	0.752	0.078	0.226
grid_3000	0.186	0.175	0.179	0.156	0.165	0.121	0.186	0.175	0.245	0.181	0.188	0.094	0.148
grid_3288	0.13	0.143	0.136	0.133	0.141	0.121	0.147	0.174	0.243	0.142	0.152	0.088	0.141
grid_3600	0.324	0.275	0.299	0.189	0.231	0.141	0.196	0.176	0.219	0.158	0.223	0.055	0.13
grid_4800	0.342	0.354	0.354	0.325	0.349	0.091	0.221	0.164	0.228	0.353	0.344	0.037	0.152
btree_129	0.305	0.429	0.402	0.409	0.354	0.378	0.535	0.724	0.764	0.443	0.47	0.352	0.489
btree_193	0.258	0.311	0.306	0.199	0.333	0.314	0.501	0.736	0.772	0.375	0.407	0.257	0.404
btree_257	0.224	0.253	0.243	0.244	0.296	0.313	0.498	0.737	0.779	0.33	0.363	0.218	0.419
btree_321	0.204	0.224	0.2	0.23	0.27	0.256	0.483	0.735	0.774	0.315	0.359	0.178	0.382
btree_385	0.176	0.205	0.183	0.135	0.24	0.223	0.471	0.719	0.754	0.332	0.309	0.17	0.313
btree_449	0.176	0.181	0.157	0.172	0.235	0.182	0.45	0.706	0.742	0.296	0.318	0.154	0.327
btree_513	0.154	0.155	0.143	0.186	0.225	0.178	0.407	0.725	0.758	0.261	0.319	0.145	0.292
btree_577	0.145	0.155	0.132	0.184	0.245	0.178	0.395	0.721	0.737	0.309	0.289	0.137	0.245
btree_705	0.137	0.126	0.117	0.164	0.211	0.154	0.37	0.695	0.737	0.239	0.261	0.122	0.222
btree_769	0.129	0.121	0.105	0.164	0.195	0.148	0.364	0.702	0.729	0.26	0.267	0.117	0.225
btree_1025	0.109	0.082	0.077	0.161	0.169	0.134	0.354	0.696	0.722	0.248	0.253	0.11	0.216
btree_1537	0.095	0.064	0.057	0.152	0.162	0.118	0.298	0.676	0.702	0.225	0.239	0.087	0.199
btree_2049	0.078	0.044	0.044	0.155	0.159	0.112	0.268	0.658	0.701	0.216	0.226	0.074	0.183
btree_3073	0.054	0.03	0.031	0.144	0.141	0.109	0.212	0.647	0.69	0.205	0.219	0.061	0.164
btree_4097	0.046	0.022	0.024	0.146	0.134	0.107	0.195	0.617	0.668	0.178	0.233	0.05	0.159
bio-CE-GT	0.392	0.302	0.316	0.189	0.191	0.355	0.343	0.43	0.417	0.337	0.341	0.323	0.343
bio-CE-HT	0.185	0.107	0.107	0.162	0.15	0.175	0.129	0.415	0.477	0.174	0.151	0.091	0.189
bio-celegans	0.639	0.615	0.611	0.433	0.367	0.542	0.527	0.515	0.497	0.513	0.527	0.494	0.534
bio-yeast	0.258	0.153	0.155	0.175	0.192	0.203	0.166	0.407	0.468	0.208	0.19	0.119	0.214
bus494	0.359	0.307	0.303	0.265	0.313	0.279	0.399	0.472	0.567	0.37	0.371	0.205	0.318
bus662	0.337	0.27	0.273	0.207	0.299	0.236	0.326	0.4	0.458	0.365	0.366	0.13	0.28
bus685	0.437	0.444	0.446	0.321	0.393	0.38	0.483	0.446	0.499	0.461	0.495	0.243	0.43
bus1138	0.295	0.261	0.262	0.212	0.307	0.236	0.335	0.441	0.54	0.34	0.332	0.117	0.28
ca-CSphd	0.489	0.391	0.39	0.423	0.345	0.464	0.414	0.566	0.629	0.512	0.468	0.231	0.507
ca-Erdos992	0.279	0.17	0.172	0.263	0.211	0.279	0.086	0.424	0.452	0.261	0.173	0.084	0.27
CA-GrQc	0.149	0.092	0.095	0.052	0.068	0.138	0.116	0.317	0.398	0.131	0.129	0.111	0.14
fb-pages-food	0.472	0.409	0.4	0.274	0.326	0.46	0.457	0.537	0.547	0.442	0.441	0.42	0.447
soc-wiki-Vote	0.289	0.248	0.25	0.17	0.162	0.245	0.233	0.286	0.283	0.235	0.237	0.228	0.243
us_powergrid	0.189	0.145	0.145	0.126	0.286	0.1	0.213	0.363	0.469	0.293	0.314	0.053	0.191
visbrazil	0.457	0.385	0.388	0.345	0.374	0.468	0.452	0.505	0.518	0.41	0.408	0.38	0.422

Table 7. Normalized crosslessness of 13 layout methods on different graphs.

	BSM	SM	SM-F	Maxent	Maxent-F	FDP	FDP-F	LinLog	LinLog-F	FM^3	FM^3-F	FDP-SGD	FDP-BH
cluster_198	0.781	0.755	0.756	0.719	0.717	0.764	0.766	0.81	0.803	0.842	0.84	0.822	0.855
cluster_378	0.802	0.858	0.836	0.84	0.85	0.823	0.863	0.877	0.889	0.861	0.848	0.838	0.832
cluster_596	0.794	0.774	0.777	0.744	0.745	0.805	0.803	0.857	0.862	0.792	0.784	0.797	0.816
cluster_800	0.765	0.749	0.749	0.677	0.671	0.755	0.766	0.838	0.835	0.755	0.756	0.765	0.766
cluster_991	0.818	0.795	0.796	0.751	0.764	0.831	0.822	0.873	0.88	0.815	0.803	0.828	0.818
cluster_1197	0.806	0.771	0.776	0.721	0.733	0.803	0.794	0.861	0.869	0.798	0.792	0.788	0.812
cluster_1485	0.818	0.808	0.803	0.787	0.803	0.831	0.829	0.886	0.886	0.827	0.82	0.82	0.829
cluster_1983	0.814	0.789	0.781	0.727	0.743	0.804	0.819	0.882	0.881	0.807	0.81	0.821	0.814
cluster_1998	0.813	0.793	0.807	0.764	0.783	0.837	0.828	0.884	0.884	0.816	0.815	0.84	0.824
cluster_2998	0.821	0.772	0.782	0.728	0.741	0.828	0.815	0.892	0.89	0.812	0.812	0.825	0.824
cluster_3486	0.823	0.793	0.798	0.78	0.826	0.85	0.858	0.897	0.896	0.843	0.825	0.848	0.858
cluster_3995	0.827	0.788	0.784	0.753	0.766	0.836	0.836	0.898	0.896	0.83	0.827	0.834	0.835
cluster_4463	0.816	0.812	0.802	0.781	0.838	0.842	0.856	0.898	0.901	0.839	0.834	0.845	0.849
cluster_4962	0.825	0.775	0.783	0.748	0.769	0.837	0.849	0.904	0.899	0.827	0.833	0.844	0.84
cluster_4999	0.823	0.805	0.799	0.783	0.83	0.853	0.848	0.89	0.895	0.834	0.843	0.844	0.849
grid_100	0.992	1	0.821	0.771	0.973	1	0.761	0.883	0.946	0.768	0.989	0.817	1
grid_289	0.981	0.86	1	0.885	0.847	0.842	0.979	0.89	0.951	0.915	0.878	0.885	0.862
grid_500	0.86	0.969	0.861	0.869	0.856	0.885	0.968	0.902	0.949	0.853	0.994	0.811	0.846
grid_800	0.881	0.894	0.889	0.907	0.902	0.836	0.924	0.885	0.897	0.877	0.867	0.888	0.886
grid_1000	0.915	0.885	0.916	0.905	0.917	0.896	0.928	0.902	0.859	0.902	0.851	0.882	0.844
grid_1200	0.918	0.903	0.916	0.902	0.881	0.828	0.904	0.896	0.897	0.898	0.854	0.867	0.871
grid_1500	0.896	0.966	0.857	0.901	0.905	0.919	0.93	0.872	0.868	0.835	0.894	0.856	0.85
grid_1600	0.889	0.937	0.895	0.954	0.975	0.893	0.918	0.907	0.897	0.997	0.892	0.886	0.892
grid_2000	0.832	0.872	0.899	0.925	0.868	0.828	0.87	0.869	0.918	0.873	0.909	0.84	0.893
grid_2250	0.901	0.969	0.936	0.797	0.879	0.858	0.874	0.895	0.945	0.895	0.915	0.84	0.855
grid_2500	0.945	0.955	0.976	0.949	0.974	0.884	0.927	0.896	0.893	0.997	0.897	0.861	0.886
grid_3000	0.974	0.982	0.937	0.862	0.979	0.841	0.851	0.89	0.898	0.86	0.855	0.855	0.872
grid_3288	0.892	0.879	0.895	0.845	0.907	0.842	0.88	0.88	0.885	0.889	0.895	0.855	0.862
grid_3600	0.929	0.932	0.931	0.932	0.908	0.864	0.911	0.903	0.9	0.886	0.903	0.881	0.9
grid_4800	0.94	0.954	0.953	0.959	0.951	0.844	0.899	0.912	0.908	0.91	0.912	0.886	0.919
btree_129	0.94	0.951	0.953	0.959	0.951	0.812	0.894	0.881	0.908	0.83	0.837	0.817	0.831
btree_193	0.847	0.854	0.854	0.825	0.809	0.822	0.845	0.854	0.862	0.853	0.859	0.802	0.857
btree_257	0.835	0.859	0.855	0.802	0.832	0.815	0.881	0.872	0.894	0.861	0.855	0.806	0.862
btree_321	0.813	0.85	0.846	0.835	0.856	0.828	0.873	0.864	0.901	0.849	0.866	0.804	0.859
btree_385	0.834	0.829	0.845	0.848	0.88	0.819	0.876	0.884	0.89	0.874	0.862	0.793	0.857
btree_449	0.811	0.861	0.848	0.832	0.881	0.82	0.885	0.872	0.898	0.852	0.872	0.823	0.876
btree_513	0.797	0.842	0.829	0.869	0.879	0.811	0.887	0.867	0.888	0.873	0.892	0.803	0.862
btree_577	0.803	0.848	0.846	0.866	0.854	0.812	0.884	0.878	0.876	0.879	0.87	0.819	0.851
btree_705	0.797	0.853	0.85	0.844	0.881	0.82	0.871	0.874	0.877	0.882	0.872	0.806	0.855
btree_769	0.849	0.846	0.847	0.848	0.884	0.816	0.892	0.878	0.881	0.891	0.883	0.801	0.859
btree_1025	0.832	0.846	0.853	0.829	0.869	0.818	0.889	0.883	0.877	0.88	0.876	0.832	0.871
btree_1537	0.835	0.839	0.828	0.875	0.853	0.818	0.91	0.868	0.885	0.9	0.904	0.82	0.882
btree_2049	0.817	0.841	0.846	0.86	0.874	0.82	0.893	0.87	0.87	0.907	0.914	0.806	0.883
btree_3073	0.817	0.827	0.839	0.865	0.914	0.819	0.89	0.871	0.85	0.924	0.924	0.807	0.878
btree_4097	0.828	0.841	0.836	0.857	0.922	0.824	0.904	0.867	0.839	0.916	0.929	0.812	0.889
bio-CE-GT	0.726	0.75	0.74	0.703	0.703	0.77	0.772	0.788	0.799	0.757	0.749	0.77	0.78
bio-CE-HT	0.799	0.817	0.808	0.792	0.835	0.826	0.812	0.849	0.857	0.824	0.83	0.802	0.833
bio-celegans	0.671	0.699	0.709	0.675	0.569	0.7	0.713	0.769	0.739	0.693	0.711	0.722	0.711
bio-yeast	0.805	0.798	0.838	0.804	0.831	0.821	0.809	0.883	0.869	0.821	0.822	0.798	0.828
bus494	0.871	0.865	0.869	0.867	0.888	0.849	0.887	0.915	0.92	0.884	0.883	0.84	0.875
bus662	0.879	0.863	0.896	0.86	0.885	0.862	0.892	0.903	0.9	0.874	0.887	0.824	0.881
bus685	0.887	0.907	0.901	0.865	0.907	0.883	0.893	0.92	0.935	0.906	0.899	0.872	0.896
bus1138	0.894	0.879	0.897	0.9	0.9	0.892	0.898	0.913	0.906	0.89	0.888	0.84	0.909
ca-CSphd	0.87	0.867	0.882	0.869	0.871	0.87	0.884	0.883	0.913	0.901	0.901	0.834	0.883
ca-Erdos992	0.754	0.753	0.766	0.741	0.827	0.81	0.758	0.849	0.828	0.803	0.803	0.771	0.795
CA-GrQc	0.837	0.826	0.816	0.804	0.799	0.845	0.821	0.876	0.891	0.829	0.825	0.842	0.826
fb-pages-food	0.748	0.75	0.763	0.75	0.731	0.779	0.796	0.829	0.785	0.777	0.786	0.731	0.793
soc-wiki-Vote	0.736	0.805	0.772	0.762	0.758	0.821	0.776	0.804	0.797	0.787	0.823	0.822	0.721
us_powergrid	0.907	0.917	0.887	0.916	0.943	0.882	0.903	0.912	0.908	0.924	0.927	0.849	0.897
visbrazil	0.816	0.834	0.824	0.775	0.825	0.818	0.866	0.877	0.877	0.83	0.834	0.84	0.855

Table 8. Normalized average min angle of 13 layout methods on different graphs.

	BSM	SM	SM-F	Maxent	Maxent-F	FDP	FDP-F	LinLog	LinLog-F	FM^3	FM^3-F	FDP-SGD	FDP-BH
cluster_198	0.328	0.344	0.342	0.309	0.3	0.343	0.339	0.463	0.392	0.382	0.381	0.596	0.622
cluster_378	0.512	0.554	0.548	0.509	0.505	0.541	0.57	0.635	0.625	0.445	0.455	0.517	0.541
cluster_596	0.227	0.26	0.257	0.218	0.222	0.247	0.229	0.334	0.289	0.747	0.752	0.234	0.258
cluster_800	0.105	0.127	0.124	0.101	0.094	0.115	0.103	0.162	0.136	0.889	0.888	0.114	0.11
cluster_991	0.223	0.25	0.249	0.222	0.219	0.242	0.229	0.341	0.293	0.769	0.765	0.227	0.254
cluster_1197	0.147	0.171	0.174	0.152	0.145	0.159	0.161	0.241	0.194	0.837	0.838	0.158	0.174
cluster_1485	0.246	0.273	0.288	0.253	0.256	0.276	0.276	0.378	0.341	0.721	0.72	0.255	0.295
cluster_1983	0.131	0.156	0.158	0.137	0.136	0.14	0.135	0.211	0.173	0.855	0.855	0.133	0.155
cluster_1998	0.212	0.229	0.237	0.212	0.212	0.228	0.225	0.332	0.288	0.772	0.778	0.212	0.247
cluster_2998	0.107	0.123	0.121	0.115	0.108	0.116	0.109	0.173	0.144	0.886	0.887	0.112	0.12
cluster_3486	0.17	0.198	0.198	0.186	0.187	0.188	0.193	0.283	0.231	0.8	0.802	0.173	0.206
cluster_3995	0.114	0.132	0.131	0.122	0.119	0.121	0.121	0.189	0.152	0.875	0.874	0.111	0.13
cluster_4463	0.2	0.232	0.232	0.216	0.216	0.227	0.231	0.328	0.283	0.766	0.767	0.208	0.243
cluster_4962	0.099	0.116	0.117	0.106	0.104	0.108	0.11	0.164	0.131	0.89	0.888	0.102	0.118
cluster_4999	0.204	0.235	0.233	0.216	0.221	0.232	0.23	0.332	0.279	0.761	0.768	0.206	0.244
grid_100	0.875	0.894	0.892	0.889	0.896	0.822	0.831	0.844	0.845	0.261	0.259	0.711	0.855
grid_289	0.904	0.931	0.929	0.922	0.933	0.567	0.846	0.547	0.775	0.196	0.171	0.458	0.57
grid_500	0.914	0.943	0.94	0.93	0.945	0.636	0.856	0.487	0.602	0.196	0.155	0.482	0.775
grid_800	0.671	0.564	0.565	0.377	0.367	0.518	0.58	0.468	0.535	0.434	0.509	0.454	0.631
grid_1000	0.666	0.532	0.531	0.353	0.357	0.464	0.534	0.433	0.52	0.466	0.481	0.486	0.447
grid_1200	0.659	0.505	0.505	0.515	0.394	0.424	0.48	0.437	0.487	0.507	0.513	0.463	0.552
grid_1500	0.245	0.279	0.283	0.176	0.219	0.452	0.137	0.311	0.517	0.847	0.867	0.317	0.775
grid_1600	0.927	0.961	0.959	0.947	0.936	0.488	0.746	0.392	0.588	0.162	0.185	0.433	0.615
grid_2000	0.239	0.26	0.267	0.05	0.105	0.368	0.157	0.233	0.52	0.857	0.867	0.262	0.414
grid_2250	0.241	0.332	0.296	0.119	0.24	0.313	0.268	0.298	0.492	0.546	0.597	0.281	0.622
grid_2500	0.929	0.966	0.963	0.942	0.918	0.44	0.729	0.401	0.529	0.139	0.168	0.417	0.541
grid_3000	0.232	0.277	0.287	0.096	0.213	0.339	0.269	0.256	0.43	0.798	0.707	0.325	0.37
grid_3288	0.267	0.698	0.529	0.703	0.648	0.416	0.243	0.296	0.493	0.634	0.629	0.309	0.346
grid_3600	0.499	0.517	0.51	0.711	0.54	0.44	0.537	0.401	0.447	0.217	0.422	0.423	0.49
grid_4800	0.837	0.93	0.926	0.906	0.915	0.457	0.441	0.42	0.477	0.187	0.209	0.457	0.542
bbtree_129	0.589	0.69	0.706	0.65	0.757	0.701	0.715	0.858	0.863	0.32	0.304	0.63	0.679
bbtree_193	0.561	0.678	0.664	0.663	0.752	0.683	0.678	0.849	0.842	0.34	0.328	0.613	0.648
bbtree_257	0.558	0.666	0.656	0.651	0.742	0.665	0.67	0.847	0.875	0.359	0.335	0.616	0.656
bbtree_321	0.545	0.652	0.653	0.627	0.725	0.665	0.667	0.851	0.872	0.374	0.345	0.607	0.644
bbtree_385	0.545	0.646	0.639	0.663	0.746	0.661	0.65	0.827	0.841	0.358	0.364	0.608	0.635
bbtree_449	0.541	0.645	0.635	0.622	0.737	0.653	0.65	0.826	0.856	0.365	0.383	0.601	0.621
bbtree_513	0.539	0.643	0.627	0.606	0.727	0.646	0.648	0.808	0.852	0.374	0.363	0.606	0.617
bbtree_577	0.545	0.635	0.627	0.601	0.743	0.645	0.63	0.811	0.861	0.354	0.371	0.598	0.605
bbtree_705	0.547	0.632	0.626	0.602	0.723	0.625	0.605	0.799	0.864	0.376	0.389	0.609	0.604
bbtree_769	0.544	0.619	0.61	0.595	0.727	0.634	0.614	0.807	0.852	0.375	0.38	0.595	0.6
bbtree_1025	0.537	0.617	0.599	0.575	0.722	0.63	0.611	0.806	0.859	0.388	0.397	0.599	0.59
bbtree_1537	0.533	0.598	0.588	0.555	0.713	0.63	0.587	0.807	0.85	0.396	0.401	0.593	0.581
bbtree_2049	0.538	0.591	0.577	0.531	0.709	0.619	0.585	0.804	0.846	0.403	0.4	0.595	0.569
bbtree_3073	0.538	0.577	0.571	0.527	0.695	0.616	0.564	0.799	0.842	0.414	0.415	0.59	0.561
bbtree_4097	0.545	0.574	0.563	0.516	0.697	0.611	0.559	0.798	0.833	0.426	0.419	0.592	0.558
bio-CE-GT	0.197	0.21	0.211	0.236	0.22	0.206	0.208	0.379	0.222	0.79	0.799	0.201	0.2
bio-CE-HT	0.695	0.748	0.745	0.724	0.766	0.753	0.73	0.802	0.815	0.251	0.261	0.74	0.749
bio-celegans	0.097	0.109	0.102	0.134	0.097	0.125	0.142	0.217	0.086	0.889	0.881	0.115	0.12
bio-yeast	0.669	0.714	0.718	0.694	0.742	0.732	0.707	0.771	0.775	0.276	0.29	0.71	0.722
bus494	0.651	0.713	0.715	0.656	0.661	0.698	0.707	0.767	0.778	0.302	0.299	0.683	0.693
bus662	0.591	0.64	0.646	0.599	0.636	0.664	0.668	0.719	0.727	0.345	0.349	0.632	0.668
bus685	0.379	0.437	0.431	0.369	0.382	0.419	0.431	0.446	0.454	0.571	0.579	0.367	0.411
bus1138	0.602	0.662	0.656	0.616	0.624	0.656	0.646	0.723	0.733	0.365	0.369	0.644	0.644
ca-CSphd	0.819	0.849	0.842	0.829	0.865	0.846	0.81	0.891	0.892	0.158	0.161	0.83	0.84
ca-Erdos992	0.759	0.773	0.772	0.776	0.835	0.825	0.795	0.867	0.863	0.184	0.189	0.809	0.803
CA-GrQc	0.31	0.327	0.325	0.374	0.373	0.374	0.328	0.414	0.379	0.636	0.647	0.338	0.362
fb-pages-food	0.312	0.338	0.336	0.346	0.335	0.334	0.334	0.426	0.327	0.654	0.669	0.324	0.335
soc-wiki-Vote	0.336	0.353	0.361	0.38	0.362	0.377	0.37	0.468	0.381	0.624	0.628	0.361	0.368
us_powergrid	0.561	0.604	0.601	0.58	0.63	0.638	0.587	0.704	0.711	0.38	0.387	0.623	0.614
visbrazil	0.559	0.585	0.573	0.558	0.525	0.592	0.594	0.672	0.617	0.418	0.423	0.599	0.581

Table 9. Normalized cluster extraction of 13 layout methods on different graphs.

	BSM	SM	SM-F	Maxent	Maxent-F	FDP	FDP-F	LinLog	LinLog-F	FM^3	FM^3-F	FDP-SGD	FDP-BH
cluster_198	0.8	0.795	0.786	0.783	0.79	0.796	0.816	0.852	0.861	0.781	0.776	0.815	0.861
cluster_378	0.858	0.858	0.86	0.877	0.909	0.815	0.913	0.9	0.943	0.857	0.856	0.869	0.847
cluster_596	0.852	0.829	0.829	0.829	0.816	0.864	0.869	0.939	0.946	0.849	0.853	0.879	0.874
cluster_800	0.844	0.822	0.823	0.786	0.809	0.864	0.87	0.969	0.967	0.854	0.851	0.863	0.87
cluster_991	0.856	0.829	0.827	0.853	0.865	0.892	0.893	0.95	0.965	0.87	0.869	0.897	0.884
cluster_1197	0.883	0.831	0.841	0.821	0.847	0.889	0.892	0.969	0.974	0.876	0.867	0.892	0.902
cluster_1485	0.868	0.846	0.844	0.867	0.892	0.889	0.901	0.965	0.969	0.88	0.876	0.904	0.893
cluster_1983	0.873	0.834	0.843	0.848	0.865	0.898	0.89	0.959	0.969	0.892	0.886	0.913	0.901
cluster_1998	0.876	0.84	0.843	0.833	0.848	0.9	0.902	0.98	0.981	0.882	0.87	0.907	0.89
cluster_2998	0.891	0.848	0.844	0.841	0.852	0.913	0.911	0.983	0.984	0.898	0.897	0.919	0.909
cluster_3486	0.883	0.856	0.861	0.875	0.925	0.925	0.934	0.978	0.981	0.908	0.899	0.934	0.935
cluster_3995	0.893	0.851	0.845	0.859	0.882	0.92	0.92	0.982	0.986	0.909	0.903	0.93	0.919
cluster_4463	0.883	0.852	0.857	0.868	0.921	0.914	0.929	0.97	0.974	0.903	0.893	0.927	0.925
cluster_4962	0.883	0.854	0.871	0.893	0.944	0.914	0.938	0.969	0.971	0.907	0.909	0.935	0.921
cluster_4999	0.901	0.851	0.856	0.855	0.871	0.923	0.932	0.984	0.989	0.904	0.898	0.926	0.929

Table 10. Normalized cluster distance of 13 layout methods on different graphs.

	BSM	SM	SM-F	Maxent	Maxent-F	FDP	FDP-F	LinLog	LinLog-F	FM^3	FM^3-F	FDP-SGD	FDP-BH
cluster_198	0.067	0.03	0.057	0.041	0.047	0.059	0.059	0.102	0.084	0.07	0.06	0.066	0.089
cluster_378	0.119	0.109	0.107	0.088	0.096	0.069	0.099	0.22	0.181	0.11	0.111	0.099	0.121
cluster_596	0.058	0.018	0.029	0.024	0.028	0.04	0.053	0.145	0.145	0.042	0.045	0.046	0.052
cluster_800	0.045	0.011	0.021	0.017	0.017	0.03	0.028	0.138	0.133	0.024	0.028	0.031	0.03
cluster_991	0.059	0.018	0.027	0.018	0.022	0.054	0.047	0.119	0.101	0.05	0.046	0.054	0.06
cluster_1197	0.06	0.013	0.021	0.019	0.018	0.043	0.042	0.153	0.145	0.033	0.038	0.037	0.055
cluster_1485	0.072	0.022	0.032	0.027	0.034	0.059	0.07	0.137	0.132	0.062	0.059	0.033	0.086
cluster_1983	0.064	0.013	0.021	0.019	0.032	0.045	0.056	0.114	0.108	0.04	0.038	0.048	0.058
cluster_1998	0.06	0.011	0.016	0.013	0.015	0.043	0.046	0.141	0.15	0.058	0.057	0.048	0.071
cluster_2998	0.052	0.007	0.011	0.009	0.012	0.043	0.04	0.153	0.176	0.034	0.033	0.035	0.044
cluster_3486	0.068	0.014	0.017	0.014	0.053	0.061	0.08	0.172	0.209	0.069	0.066	0.072	0.052
cluster_3995	0.055	0.008	0.013	0.01	0.018	0.052	0.043	0.153	0.179	0.042	0.046	0.045	0.055
cluster_4463	0.066	0.014	0.018	0.015	0.04	0.071	0.067	0.154	0.162	0.068	0.069	0.073	0.066
cluster_4962	0.068	0.013	0.015	0.014	0.037	0.056	0.062	0.107	0.138	0.039	0.04	0.033	0.049
cluster_4999	0.048	0.005	0.009	0.009	0.015	0.045	0.048	0.187	0.182	0.065	0.067	0.061	0.063

Table 11. The detailed results of runtime(s) of different implementations of different methods on different graphs.

	BSM	SM	SM-F	Maxent	Maxent-F	FDP	FDP-F	LinLog	LinLog-F	FM^3	FM^3-F	FDP-SGD	FDP-BH
block_198	0.004	0.033	0.004	0.055	0.037	0.029	0.032	1.32	0.034	0.08	0.093	0.009	0.102
block_378	0.016	0.082	0.014	0.112	0.068	0.055	0.058	1.464	0.064	0.139	0.173	0.158	0.187
block_596	0.068	0.279	0.065	0.203	0.132	0.176	0.197	2.632	0.113	0.217	0.232	0.132	0.183
block_800	0.158	0.621	0.144	0.298	0.167	0.353	0.16	4.061	0.166	0.29	0.309	0.24	0.218
block_991	0.288	1.098	0.261	0.4	0.224	0.604	0.21	5.644	0.224	0.393	0.453	0.368	0.245
block_1197	0.446	1.682	0.422	0.493	0.286	0.926	0.434	7.202	0.287	0.45	0.481	0.532	0.301
block_1485	0.676	2.473	0.625	0.597	0.334	1.392	0.317	8.615	0.335	0.601	0.721	0.86	0.36
block_1983	1.97	6.711	1.826	1.031	0.562	3.595	0.541	16.251	0.588	0.898	0.924	1.563	0.495
block_1998	1.097	3.854	0.999	0.755	0.425	2.059	0.399	11.972	0.446	0.987	1.003	1.537	0.507
block_2998	3.237	10.486	2.9	1.339	0.744	5.582	0.737	20.429	0.757	1.366	1.478	3.757	0.792
block_3486	4.726	15.299	4.288	1.644	0.848	8.055	0.891	25.142	0.908	1.686	1.837	4.99	0.927
block_3995	6.483	20.977	6.188	1.955	1.061	10.972	1.067	31.123	1.08	1.954	2.08	6.519	1.055
block_4463	8.582	27.303	8.085	2.252	1.257	14.257	1.194	37.311	1.303	2.497	2.551	8.446	1.254
block_4962	10.961	34.673	10.359	2.542	1.446	17.964	1.441	37.311	1.549	2.49	2.636	10.598	1.395
block_4999	13.657	43.047	12.961	2.862	1.649	22.187	1.77	41.044	1.776	2.638	2.673	10.485	1.396
grid_100	0.004	0.035	0.004	0.024	0.028	0.026	0.031	0.921	0.034	0.022	0.035	0.003	0.025
grid_289	0.035	0.169	0.032	0.062	0.076	0.059	0.075	1.117	0.078	0.089	0.09	0.022	0.085
grid_500	0.13	0.465	0.099	0.095	0.106	0.102	0.123	1.822	0.124	0.175	0.189	0.08	0.119
grid_800	0.273	1.187	0.265	0.42	0.179	0.17	0.184	2.389	0.186	0.206	0.254	0.227	0.275
grid_1000	0.431	1.781	0.45	0.533	0.24	0.352	0.222	3.211	0.228	0.327	0.325	0.366	0.244
grid_1200	0.634	2.599	0.601	0.547	0.265	0.63	0.271	4.864	0.275	0.408	0.452	0.537	0.267
grid_1500	1.069	4.075	0.979	0.732	0.368	0.939	0.339	6.458	0.457	0.537	0.552	0.884	0.344
grid_1600	1.182	4.697	1.154	0.397	0.336	0.248	0.355	3.574	0.494	0.544	0.638	0.995	0.358
grid_2000	1.871	7.146	1.767	1.204	0.533	1.397	0.461	9.382	0.525	0.612	0.766	1.554	0.44
grid_2250	2.438	9.068	2.344	0.962	0.516	2.171	0.489	12.657	0.581	0.715	0.843	2.043	0.511
grid_2500	3.093	11.051	2.794	0.844	0.514	3.829	0.578	14.898	0.6	0.886	0.973	2.51	0.596
grid_3000	4.529	16.117	4.159	1.787	0.723	5.926	0.729	20.651	0.845	1.049	1.232	3.734	0.71
grid_3288	5.504	19.267	5.044	1.43	0.856	8.13	0.808	26.094	0.887	1.328	1.647	4.462	0.782
grid_3600	6.663	22.938	6.066	1.672	0.87	11.815	0.896	33.796	0.888	1.308	1.429	5.318	0.878
grid_4800	12.323	41.364	11.549	2.625	1.037	20.706	1.305	47.298	1.332	1.907	2.004	9.744	1.198
btree_129	0.008	0.044	0.006	0.082	0.072	0.034	0.046	0.92	0.059	0.044	0.031	0.005	0.038
btree_193	0.015	0.077	0.014	0.144	0.089	0.057	0.065	1.623	0.065	0.074	0.06	0.01	0.056
btree_257	0.027	0.121	0.025	0.195	0.183	0.081	0.081	1.73	0.089	0.098	0.097	0.017	0.074
btree_321	0.045	0.181	0.038	0.227	0.127	0.112	0.095	1.899	0.106	0.124	0.134	0.027	0.091
btree_385	0.062	0.257	0.056	0.198	0.164	0.154	0.109	2.278	0.131	0.159	0.169	0.044	0.108
btree_449	0.083	0.343	0.08	0.296	0.206	0.21	0.121	2.71	0.143	0.198	0.191	0.058	0.126
btree_513	0.109	0.45	0.102	0.351	0.202	0.259	0.145	3.253	0.158	0.198	0.188	0.133	0.161
btree_577	0.144	0.565	0.132	0.396	0.236	0.33	0.144	3.421	0.176	0.221	0.215	0.132	0.148
btree_705	0.279	0.836	0.328	0.49	0.304	0.493	0.185	4.892	0.225	0.252	0.246	0.169	0.18
btree_769	0.263	0.99	0.244	0.471	0.32	0.559	0.205	4.998	0.247	0.29	0.27	0.21	0.199
btree_1025	0.489	1.77	0.442	0.65	0.533	0.98	0.4	6.312	0.328	0.445	0.425	0.388	0.404
btree_1537	1.152	3.93	1.058	0.873	0.71	2.161	0.389	10.287	0.498	0.678	0.619	0.916	0.38
btree_2049	2.149	6.964	1.916	1.327	0.949	3.832	0.608	14.869	0.649	0.989	0.902	1.682	0.509
btree_3073	5.069	15.58	4.798	1.873	1.622	8.578	0.9	24.901	1.106	1.383	1.213	3.997	0.819
btree_4097	9.347	27.836	8.762	2.449	2.209	15.239	1.249	33.732	1.493	2.053	1.81	7.106	1.11
bio-CE-GT	0.353	1.418	0.323	0.511	0.311	1.01	0.233	4.51	0.247	0.307	0.361	0.276	0.224
bio-CE-HT	2.473	8.59	2.197	1.642	2.058	5.099	0.671	29.034	0.685	1.208	1.235	1.931	0.578
bio-celegans	0.189	26.537	0.094	0.256	0.203	0.198	0.154	0.642	0.227	0.184	0.217	0.068	0.129
bio-yeast	1.179	3.815	1.287	1.05	1.327	2.147	0.677	10.247	1.183	0.755	0.788	0.824	0.376
bus494	0.109	0.484	0.097	0.284	0.196	0.218	0.166	1.814	0.162	0.266	0.274	0.082	0.138
bus662	0.192	0.802	0.18	0.37	0.314	0.478	0.168	3.098	0.202	0.271	0.306	0.143	0.172
bus685	0.241	0.86	0.192	0.379	0.289	0.525	0.22	2.998	0.206	0.239	0.232	0.157	0.196
bus1138	0.605	2.368	0.542	0.82	0.291	1.622	0.284	7.869	0.317	0.513	0.524	0.476	0.283
ca-CSphd	0.578	1.893	0.59	0.722	0.526	1.012	0.558	4.869	0.676	0.426	0.415	0.389	0.266
ca-Erdos992	13.684	45.099	13.115	3.304	2.78	29.331	1.578	48.919	1.634	2.11	2.157	10.513	1.412
CA-GrQc	9.145	32.037	8.659	2.735	1.259	16.227	1.29	56.598	1.42	2.192	2.301	7.337	1.248
fb-pages-food	0.168	0.717	0.159	0.377	0.185	0.415	0.182	2.213	0.188	0.248	0.262	0.129	0.17
soc-wiki-Vote	0.358	1.469	0.322	0.493	0.246	0.873	0.234	2.792	0.242	0.455	0.488	0.287	0.227
us_powergrid	13.232	43.782	14.531	3.088	1.371	28.942	1.509	62.763	1.768	2.525	2.51	10.325	1.344
visbrazil	0.021	0.099	0.018	0.154	0.075	0.06	0.067	0.609	0.072	0.078	0.079	0.013	0.065

2 VISUALIZATION

We also present the layout results of all the 45 graphs generated by different methods and implementations. Figs. 1, 2, 3, and 4 show the layout results of grid graphs, binary tree graphs, clustered graphs and real-world graphs, respectively.

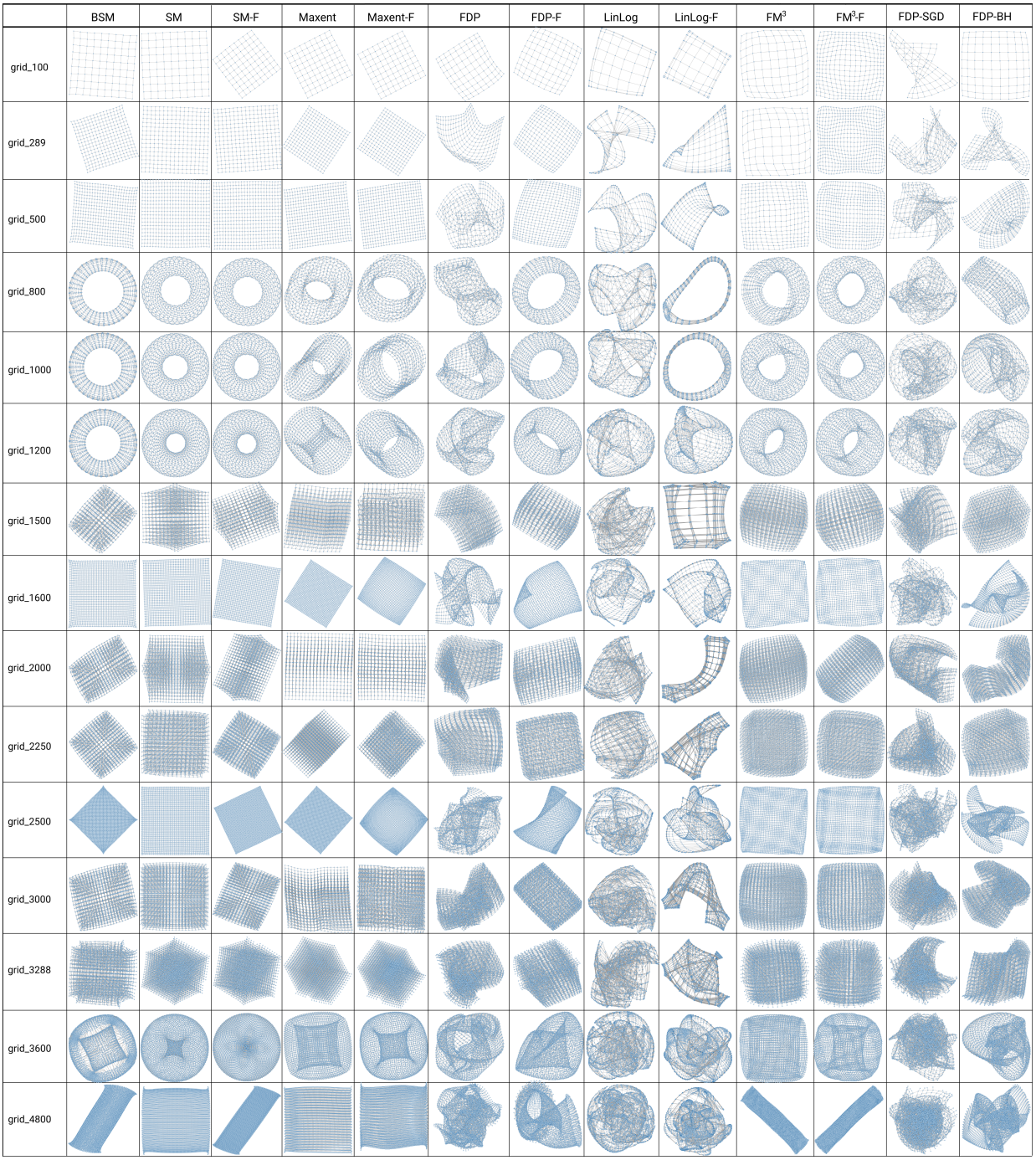

Fig. 1. Visualizations of grid graphs.

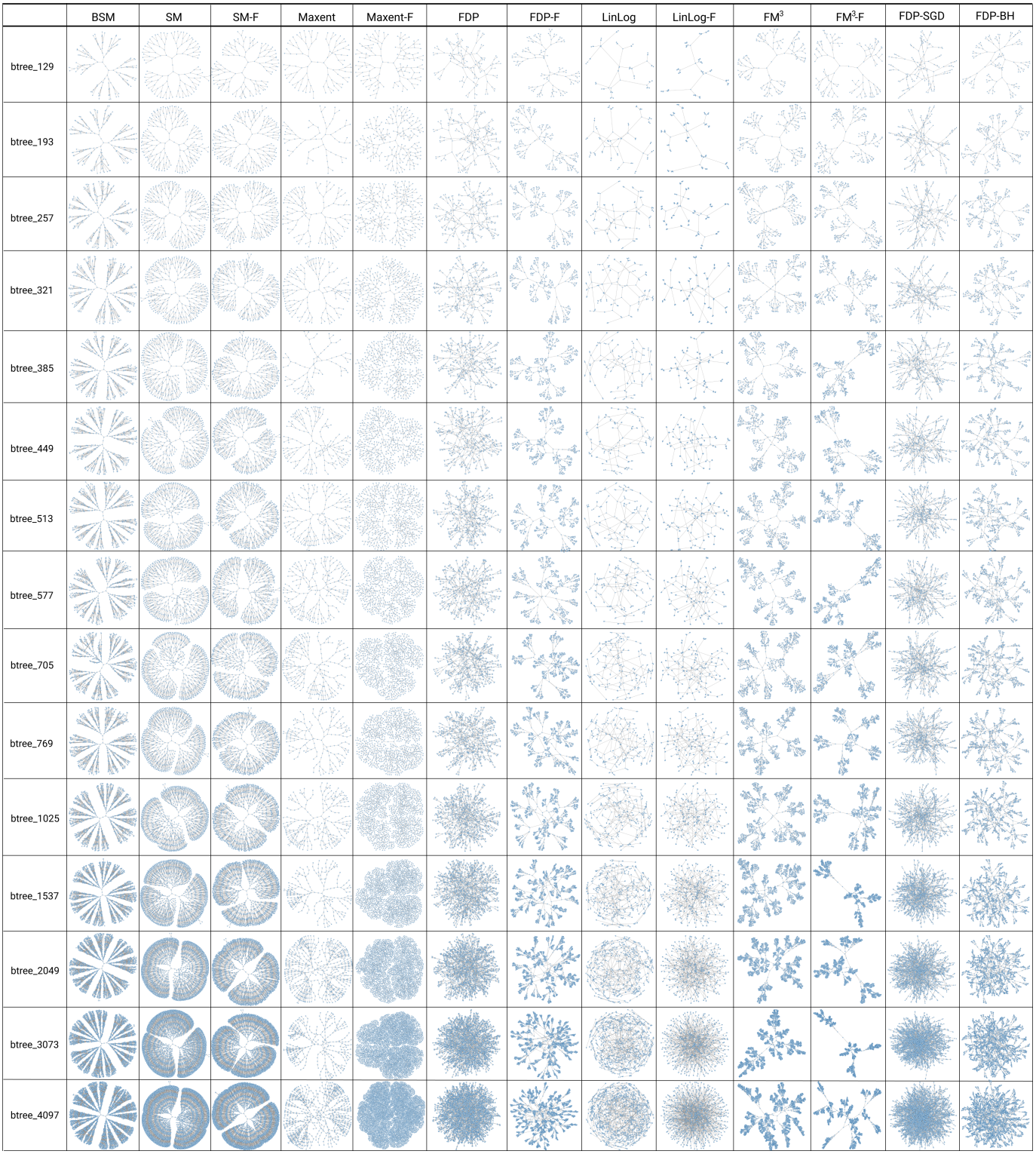

Fig. 2. Visualizations of binary tree graphs.

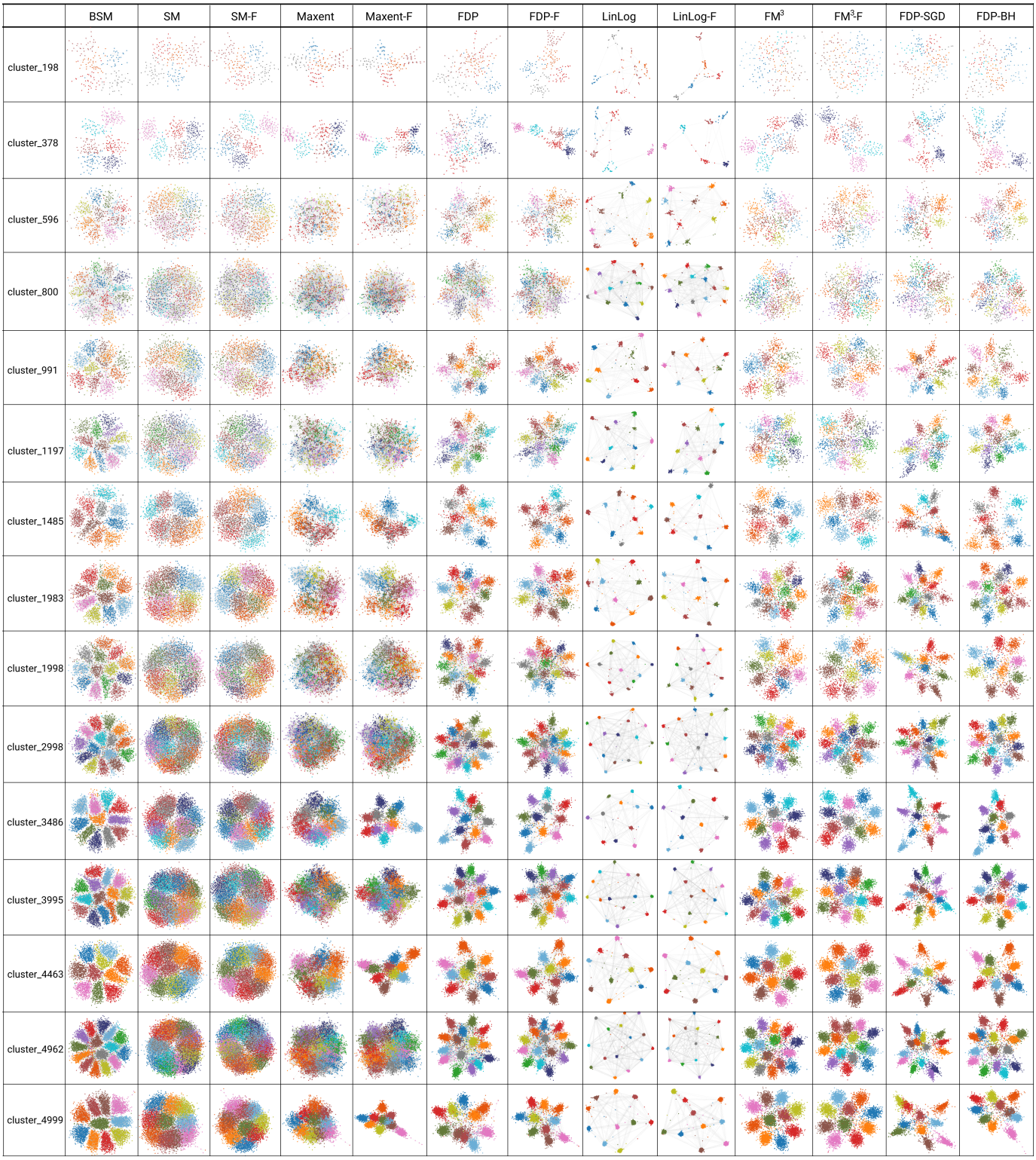

Fig. 3. Visualizations of clustered graphs.

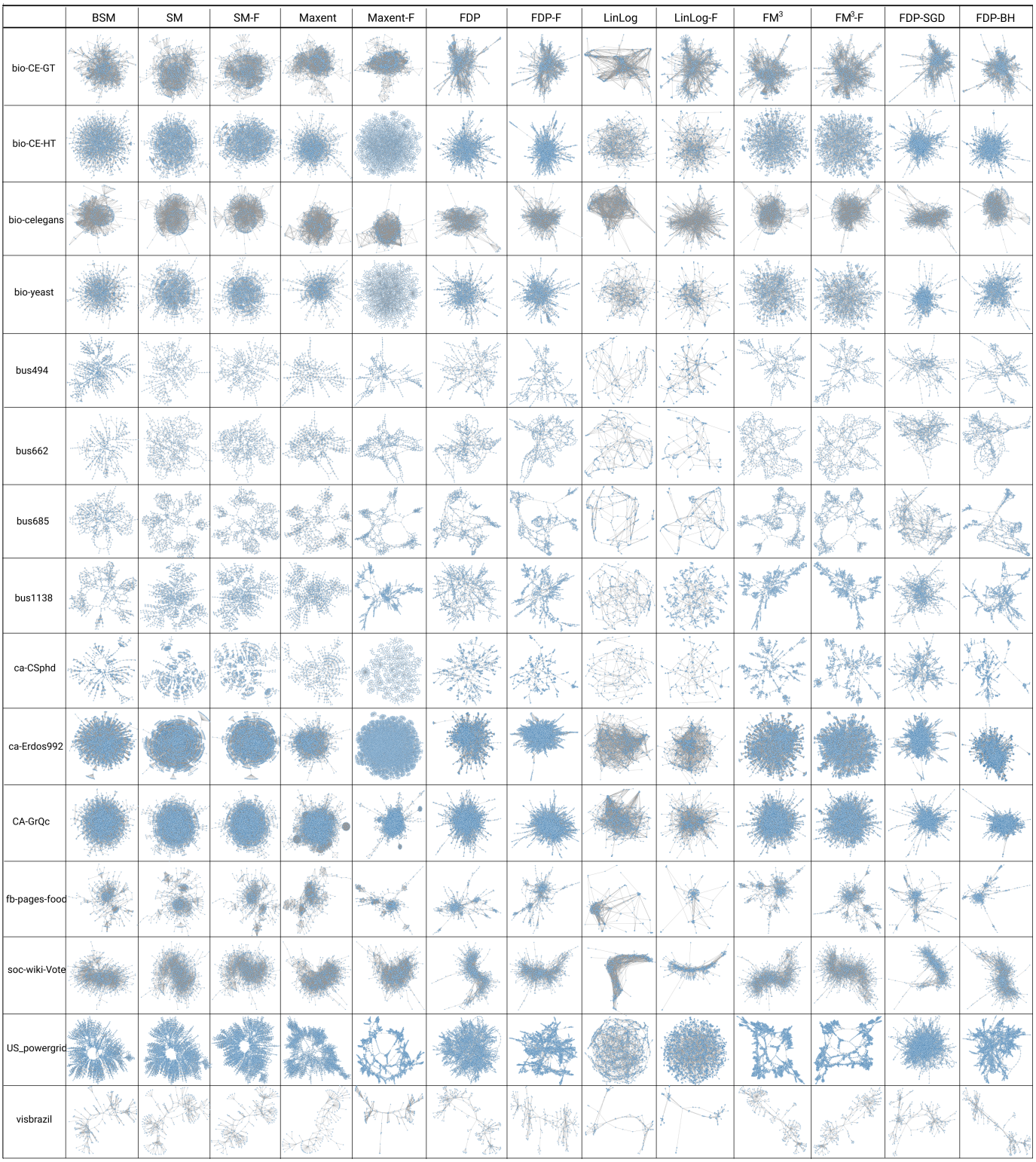

Fig. 4. Visualizations of real graphs.

3 Large Graphs

To evaluate the performance of our Augment SGD on large graphs, we collected 10 large graphs with 8,000 to 95,000 nodes from SuiteSparse Matrix Collection [1] in our experiments. Fig. 5 presents their visualization results generated by FDP-F, and Table 12 shows the node numbers, edge numbers and average runtime of 5 computations for these large graphs.

Table 12. Tested Large Graphs for evaluation

	Nodes	Edges	Runtime(s)
1.commanche_dual	7920	11881	8.211
2.pesa	11738	79566	12.086
3.ca-CondMat	21363	91286	34.228
4.tech-as-caida2007	26475	53381	34.067
5.bcsstk31	35588	572915	43.395
6.finance256	37376	130561	40.306
7.tech-internet-as	40164	85123	59.215
8.tech-p2p-gnutella	62561	147878	96.567
9.venkat50	62424	1717792	77.110
10.sc-pkustk13	94893	3260967	120.750

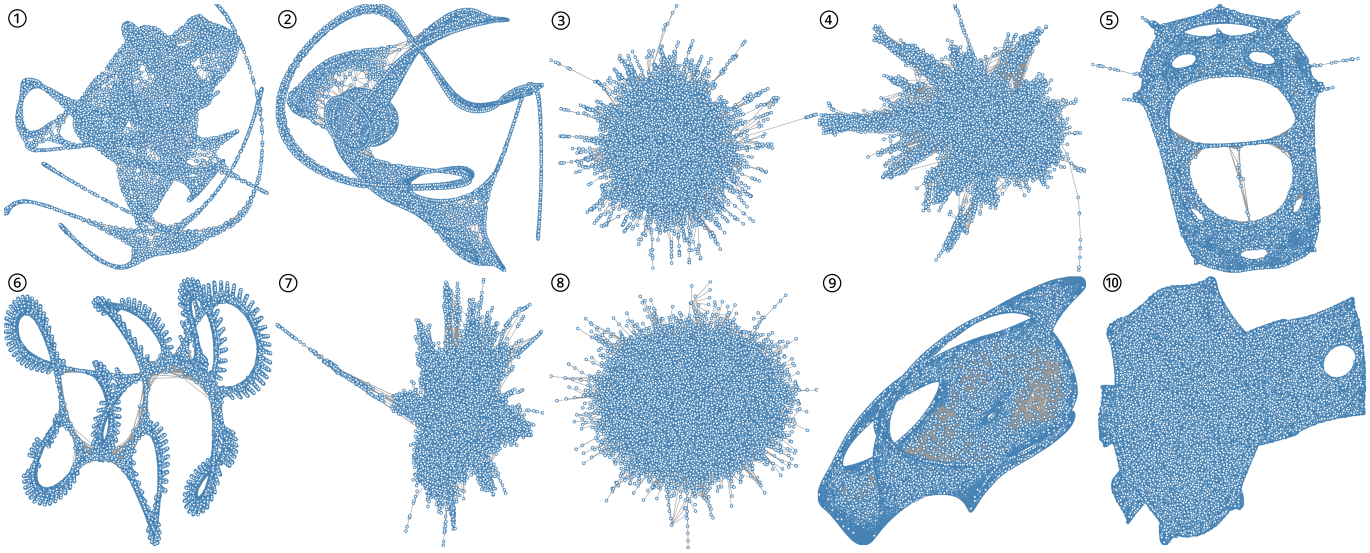

Fig. 5. Visualizations of large graphs.

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